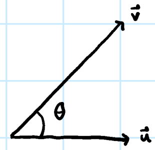
Distances from Point to Line

Wednesday, May 17, 2023 8:50 AM



reminder: if $\vec{u}, \vec{v}$ vectors then ...

- $|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \sin\theta$
- plane $\{ax + by + cz = d\}$ has normal vector $\vec{n} = (a, b, c)$

perpendicular / orthogonal

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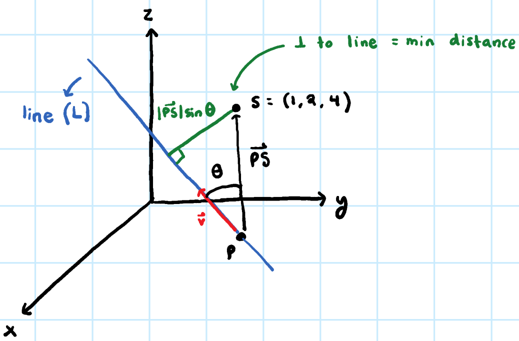
distance from S to line L is <u>min</u> of all distances (S, P), $P \in L$



P on line / determines position of L

Formula for distance from point S to line with direction $\vec{v}$ & thru point P:

$$* \quad \text{distance } (S, \text{line}) = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} \quad *$$

why ?

$$\frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} = \frac{|\vec{PS}| \cdot |\vec{v}| \sin\theta}{|\vec{v}|} = |\vec{PS}| \cdot \sin\theta$$

ex) find distance (S, L) with $S = (1, 0, 1)$ & L is intersection of $\{x = 0\}$ & $\{y - z = 0\}$

solution:

- to use equation from above, we need point P in L & direction $\vec{v}$ of L
  - for P solve $\{x = 0,\ y - z = 0\} \longrightarrow \overset{x\ y\ z}{(0, 0, 0)},\ \overset{x\ y\ z}{(0, 1, 1)}$

    either work for P

    form vectors $\langle a, b, c\rangle$

    * values in front of x, y, & z in each equation = a, b, c *
  - for $\vec{v}$: $\vec{v} = \langle 1, 0, 0\rangle \times \langle 0, 1, -1\rangle$

$$= \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \langle 0, 1, 1\rangle$$

vectors made from $x = 0$ & $y - z = 0$ are intersections (⊥) to L & $\vec{v}$ / dot product of $\vec{v}$ & other 2 vectors should = 0

with $|\vec{v}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$

- now $P = (0, 0, 0)$, $S = (1, 0, 1)$, then $\vec{PS} = \langle 1, 0, 1\rangle$

- so $\vec{PS} \times \vec{v} = \langle 1, 0, 1\rangle \times \langle 0, 1, 1\rangle$

$$= \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \langle -1, -1, 1\rangle$$

- since $|\vec{PS} \times \vec{v}| = \sqrt{(-1)^2 + (-1)^2 + 1^2} = \sqrt{3}$, distance is ...

$$\text{distance} = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} = \boxed{\frac{\sqrt{3}}{\sqrt{2}}}$$

* check by using other value of $P = (0, 1, 1)$ *